Systems of Linear Equations and Matrices

Algebra 2

Chapter 1

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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1-01 Solve Linear Systems of Equations and Inequalities by Graphing

Objectives:

- Solve a system of equations by graphing.
- Solve a system of inequalities by graphing.

1-01 Solve Linear Systems of Equations and Inequalities by Graphing

- System of equations
  - More than one equation that share the same solution.
  - Often they involve more than one variable.
  - In order to solve them, you need as many equations as there are variables.

  - $\begin{cases} 2x + 3y = 6 \\ 3x - 4y = 5 \end{cases}$

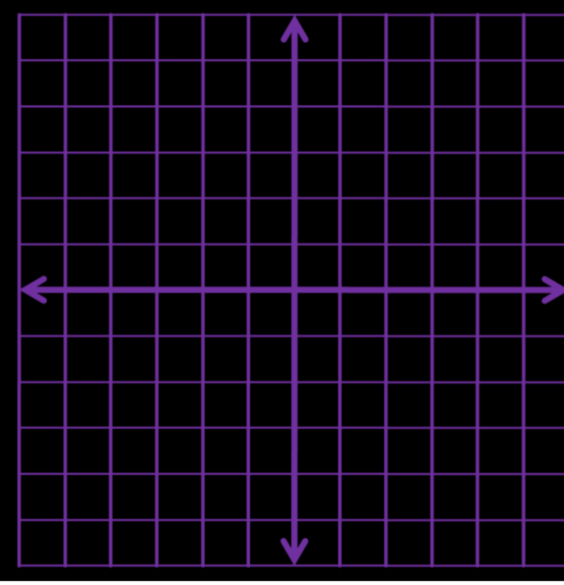
1-01 Solve Linear Systems of Equations and Inequalities by Graphing

- Solutions to systems
  - An ordered pair that works in both equations.
  - If the ordered pair works in both equations, then both graphs have to go through that point.
  - Solutions are where the graphs cross.

- Solve by graphing
  1. Graph both equations on the same graph.
  2. Where they cross is the solution.

1-01 Solve Linear Systems of Equations and Inequalities by Graphing

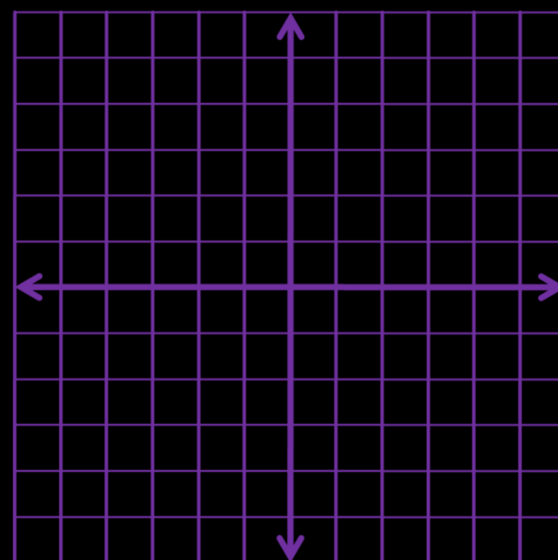
- Solve by graphing $\begin{cases} 3x + 2y = -4 \\ \ \ x + 3y = \ \ \ 1 \end{cases}$



(−2, 1)
Consistent and independent

1-01 Solve Linear Systems of Equations and Inequalities by Graphing

- Solve by graphing $\begin{cases} 3x - 2y = 10 \\ 3x - 2y = 2 \end{cases}$



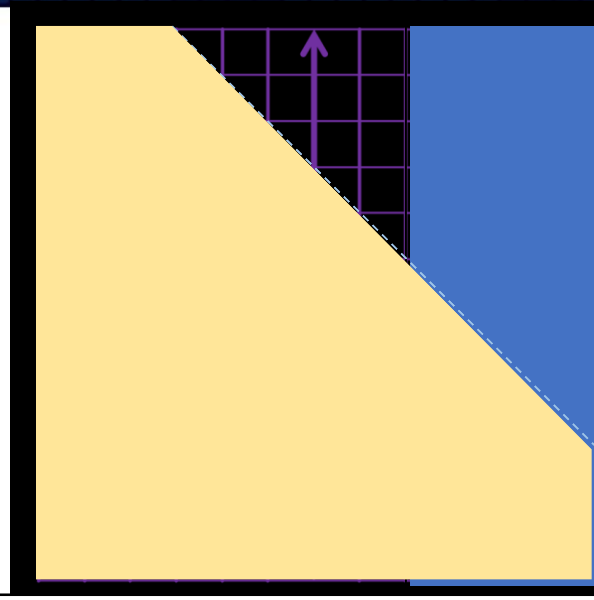
No solution
inconsistent

1-01 Solve Linear Systems of Equations and Inequalities by Graphing

- To solve systems of inequalities
    1. Graph them all on one graph.
    2. Solution is where all graphs overlap

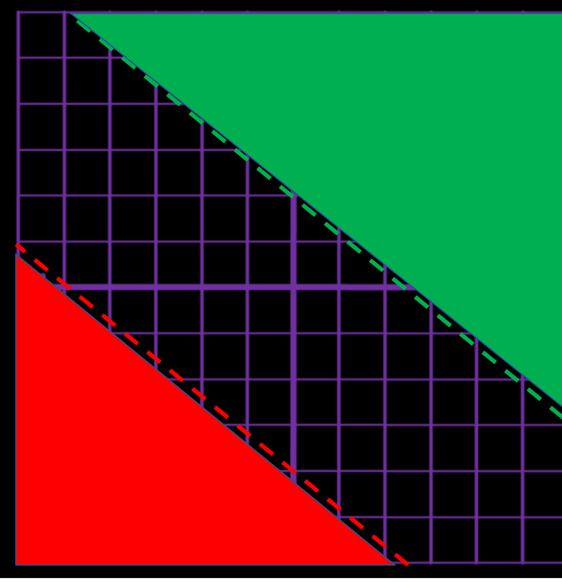
1-01 Solve Linear Systems of Equations and Inequalities by Graphing

- Solve the system of inequalities
  - $\begin{cases} x \geq 2 \\ x + y < 3 \end{cases}$



1-01 Solve Linear Systems of Equations and Inequalities by Graphing

- Solve the system of inequalities
- $\begin{cases} y < -\frac{4x}{5} - 4 \\ y > -\frac{4x}{5} + 2 \end{cases}$



1-02 Solve Linear Systems Algebraically

Objectives:

- Solve a system of equations by substitution.
- Solve a system of equations by elimination.

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1-02 Solve Linear Systems Algebraically

- Graphing to solve systems of equations has some problems.
- Can you guess some?
  - Inaccurate
  - Sometimes hard to graph

1-02 Solve Linear Systems Algebraically

- Substitution
    1. Solve one equation for one variable
    2. Use that expression to replace that variable in the other equation
    3. Solve the new equation
    4. Substitute back into the first equation
    5. Solve for the second variable

1-02 Solve Linear Systems Algebraically

$y = x + 2$
$2x + y = 8$

y is the same thing as x + 2

$y = 2 + 2$
$y = 4$

$2x + x + 2 = 8$
$3x + 2 = 8$
$3x = 6$
$x = 2$

(2,4)

1-02 Solve Linear Systems Algebraically

$$3x + 2y = 8$$
$$x + 4y = -4$$

x is by itsel
Solve fo

$$x = -4y - 4$$

$$3(-4y - 4) + 2y = 8$$
$$-12y - 12 + 2y = 8$$
$$-10y - 12 = 8$$
$$-10y = 20$$
$$y = -2$$

$$x = -4(-2) - 4$$
$$x = 4$$

$$(4, -2)$$

1-02 Solve Linear Systems Algebraically

- Elimination
  1. Line up the equations into columns
  2. Multiply one or both equations by numbers so that one variable has the same coefficient, but opposite sign
  3. Add the equations
  4. Solve the resulting equation
  5. Substitute the value into one original equation and solve

1-02 Solve Linear Systems Algebraically

$$2x - 3y = -14$$
$$(-3)(3x - y) = -7(-3)$$

$$\begin{array}{rcl} 2x - 3y &=& -14 \\ -9x + 3y &=& 21 \\ \hline -7x &=& 7 \end{array}$$

$$x = -1$$

$$2(-1) - 3y = -14$$
$$-2 - 3y = -14$$
$$-3y = -12$$
$$y = 4$$

$$(-1,4)$$

1-02 Solve Linear Systems Algebraically

$$(2)(\ \ 3x + 11y) = 4(2)$$
$$(3)(-2x - \ \ 6y) = 0(3)$$

$$\begin{aligned} 6x + 22y &= 8 \\ \underline{-6x - 18y} &\underline{= 0} \\ 4y &= 8 \\ y &= 2 \end{aligned}$$

$$-2x - 6(2) = 0$$
$$-2x - 12 = 0$$
$$-2x = 12$$
$$x = -6$$

$$(-6, 2)$$

1-02 Solve Linear Systems Algebraically

- Number of Solutions
  - If both variables disappear after you substitute or combine and
    - You get a true statement like 2 = 2 → infinite solutions
    - You get a false statement like 2 = 5 → no solution

1-02 Solve Linear Systems Algebraically

- Summary of Solving Techniques
  - When to graph?
    - To get general picture and estimate
  - When to use substitution?
    - When one of the coefficients is 1
  - When to use elimination?
    - When none of the coefficients is 1

1-03 Solve Linear Systems in Three Variables

Objectives:

- Solve a three-variable linear system of equations by elimination.
- Describe the line of intersection for many solution case of a three-variable linear system.

1-03 Solve Linear Systems in Three Variables

- We have now worked with 2 variables and 2 dimensions, but sometimes there are more

- Linear equation in 3 variables graphs a plane

1-03 Solve Linear Systems in Three Variables

- Solution to system in 3 variables
  - Ordered triple (x, y, z)

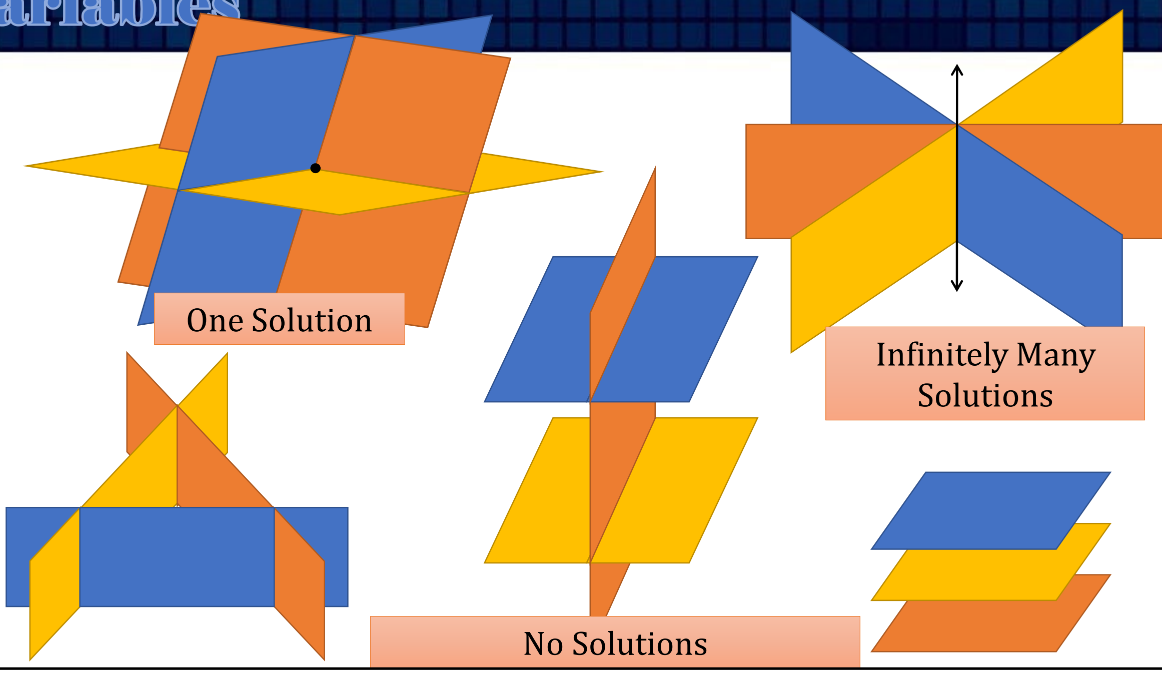
- Example: Is (2, −4, 1) a solution of
  - $\begin{cases} x + 3y - z = -11 \\ 2x + y + z = 1 \\ 5x - 2y + 3z = 21 \end{cases}$

Plug it in.

$$x + 3y - z = -11$$
$$2 + 3(-4) - 1 = -11$$
$$-11 = -11 \checkmark$$
$$2x + y + z = 1$$
$$2(2) + (-4) + 1 = 1$$
$$1 = 1 \checkmark$$
$$5x - 2y + 3z = 21$$
$$5(2) - 2(-4) + 3(1) = 21$$
$$21 = 21 \checkmark$$

Yes

1-03 Solve Linear Systems in Three Variables



1-03 Solve Linear Systems in Three Variables

- Elimination Method
- Like two variables, you just do it more than once.
  1. Combine first and second to eliminate a variable
  2. Combine second and third to eliminate the same variable as before
  3. Combine these new equations to find the two variables
  4. Substitute those two variables into one of the original equations to get the third variable

- If you get a false statement like 8 = 0 → no solution
- If you get an identity like 0 = 0 → infinitely many solutions

1-03 Solve Linear Systems in Three Variables

$$\begin{cases} 2x + 3y + 7z = -3 \\ \quad x - 6y + \quad z = -4 \\ -x - 3y + 8z = \quad 1 \end{cases}$$

Combine first two equations (multiply second by -2):
2x+3y+7z=-3
<u>-2x+12y-2z=8</u>
15y + 5z = 5 → **3y + z = 1**

Combine last two equation (just add):
x-6y+z=-4
<u>-x-3y+8z=1</u>
-9y + 9z = -3 → **-3y + 3z = -1**

Combine the combinations (just add):
3y + z = 1
<u>-3y + 3z = -1</u>
4z = 0 → **z = 0**

Substitute into 3y + z = 1:
3y + 0 = 1 → **y = 1/3**

Substitute into x – 6y + z = -4:
x – 6(1/3) + 0 = -4 → x – 2 = -4 → **x = -2**

(-2, 1/3, 0)

1-03 Solve Linear Systems in Three Variables

$$\begin{cases} -x + 2y + z = 3 \\ 2x + 2y + z = 5 \\ 4x + 4y + 2z = 6 \end{cases}$$

No Solution

1-03 Solve Linear Systems in Three Variables

$$\begin{cases} x + y + z = 6 \\ x - y + z = 6 \\ 4x + y + 4z = 24 \end{cases}$$

Combine first two equations (no multiplying necessary):

$$x + y + z = 6$$
$$x - y + z = 6$$
$$\overline{2x \quad + 2z = 12}$$
$$x + z = 6$$

Combine last two equation (just add):

$$x - y + z = 6$$
$$4x + y + 4z = 24$$
$$\overline{5x \quad + 5z = 30}$$
$$x + z = 6$$

Combine the combinations (multiply 2nd by −1):

$$x + z = 6$$
$$-x - y = -6$$
$$\overline{0 = 0}$$

This is many solutions. Describe points of intersection
Let $z = a$
Use one of the reduced equations to find *x*.

$$x + z = 6$$
$$x + a = 6$$
$$x = 6 - a$$

Substitute into one of the original equations to find *y*.

$$x + y + z = 6$$
$$(6 - a) + y + a = 6$$
$$y = 0$$

Solution is **(6 – *a*, 0, *a*)**

1-03 Solve Linear Systems in Three Variables

- If there are infinitely many solutions
  1. Let $z = a$ (Use x, y, or z based on what is convenient)
  2. Solve for y in terms of a
  3. Substitute those to find x in terms of a
  4. Sample answer $(2a, a + 4, a)$

1-03 Solve Linear Systems in T
Variables

- You have \$1.42 in quarters, nickels, and pennies. You have twice as many nickels as quarters. You have 14 coins total. How many of each coin do you have?

Translate each sentence into an equation
0.25q + 0.05n + 0.01p = 1.42
n = 2q
q + n + p = 14

This makes the system of equations.

$$\begin{cases} 0.25q + 0.05n + 0.01p = 1.42 \\ 2q - n = 0 \\ q + n + p = 14 \end{cases}$$

Start the elimination process. Because the second equation already is missing *p*, we can combine the first and third equations and eliminate *p*. Then we will have two equations with just *q* and *n*.
Multiply the first equation by −100.

$$\begin{array}{r} -25q - 5n - p = -142 \\ \underline{+q + n + p = 14} \\ -24q - 4n = -128 \end{array}$$

Combine this with the second equation. Multiply the second equation by −4.

$$-8q + 4n = 0$$
$$\underline{-24q - 4n = -128}$$
$$-32q = -128$$
$$q = 4$$

Substitute this into the second equation to find *n*.

$$2q - n = 0$$
$$2(4) - n = 0$$
$$n = 8$$

Substitute *q* = 4 and *n* = 8 into one of the original equations to find *p*.

$$q + n + p = 14$$
$$4 + 8 + p = 14$$
$$p = 2$$

Your little brother has 2 pennies, 8 nickels, and 4 quarters.

1-04 Perform Basic Matrix Operations (12.1)

Objectives:

- Solve for variables in matrices given equal matrices.
- Add and subtract matrices.
- Multiply a scalar number with a matrix.

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1-04 Perform Basic Matrix Operations (12.1)

- Matrices are simply a way to organize data.
- For example, a computer desktop wallpaper (bitmap) is a matrix. Each element tells what color pixel goes in that spot.

1-04 Perform Basic Matrix Operations (12.1)

- A matrix is a rectangular arrangement of things (variables or numbers in math)
- $\begin{bmatrix} 2 & -1 & 5 & \text{a} \\ 2 & y & 6 & b \\ 3 & 14 & x & c \end{bmatrix}$

- Dimensions
  - Rows by columns
  - 3×4 for the above matrix

1-04 Perform Basic Matrix Operations (12.1)

- In order for two matrices to be equal, they must be the same dimensions and corresponding elements must be the same
- $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

- Find the variables $\begin{bmatrix} 2 & y+1 \\ x/3 & 4 \end{bmatrix} = \begin{bmatrix} w & -4 \\ 5 & z-4 \end{bmatrix}$

$$2 = w,$$
$$\frac{x}{3} = 5 \rightarrow x = 15,$$
$$y + 1 = -4 \quad y = -5,$$
$$4 = z - 4 \rightarrow z = 8$$

1-04 Perform Basic Matrix Operations (12.1)

- Adding and Subtracting
  - You can only add and subtract matrices that are the same dimensions
  - When you add or subtract, add the corresponding elements.

- $\begin{bmatrix} 1 & 2 \\ -5 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ 4 & -3 \end{bmatrix}$

$$\begin{bmatrix} 1 + (-2) & 2 + 5 \\ -5 + 4 & 4 + (-3) \end{bmatrix}$$
$$\begin{bmatrix} -1 & 7 \\ -1 & 1 \end{bmatrix}$$

1-04 Perform Basic Matrix Operations (12.1)

- $[2 \quad -3] - [3 \quad 4] + [1 \quad 0]$

- $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 5 & 2 \end{bmatrix}$

$$[2 - 3 + 1 \quad -3 - 4 + 0]$$
$$[0 \quad -7]$$

Can’t add because different dimensions

1-04 Perform Basic Matrix Operations (12.1)

- Scalar Multiplication
  - Multiply each element by the scalar
  - Distribute
- $3\begin{bmatrix} 5 & -2 & 7 \\ -3 & 8 & 4 \end{bmatrix}$

$$\begin{bmatrix} 3\cdot 5 & 3\cdot -2 & 3\cdot 7 \\ 3\cdot -3 & 3\cdot 8 & 3\cdot 4 \end{bmatrix}$$
$$\begin{bmatrix} 15 & -6 & 21 \\ -9 & 24 & 12 \end{bmatrix}$$

1-04 Perform Basic Matrix Operations (12.1)

- The National Weather Service keeps track of weather.

June 2014	Benton Harbor	South Bend
Precip Days	13	18
Clear Days	16	13
Ab Norm T	12	19

July 2014	Benton Harbor	South Bend
Precip Days	14	15
Clear Days	18	18
Ab Norm T	2	8

- What is meaning of the first matrix + second matrix?
- Use matrix operations to find the total weather stats of each city

*Precip Days = Days with precipitation
Clear Days = Days with no clouds
Ab Norm T = Days with Above Normal Temperature

the total number of days of each type for each city

$$\begin{bmatrix} 13+14 & 18+15 \\ 16+18 & 13+18 \\ 12+2 & 19+8 \end{bmatrix}$$
$$\begin{bmatrix} 27 & 33 \\ 34 & 31 \\ 14 & 27 \end{bmatrix}$$

1-05 Multiply Matrices (12.2)

Objectives:

- Multiply matrices.
- Use matrix multiplication to solve real-life problems.

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1-05 Multiply Matrices (12.2)

- Yesterday we learned all about matrices and how to add and subtract them. But how do you multiply or divide matrices?

- Today we will multiply matrices.

- Later we will find out that you can't divide by a matrix.

1-05 Multiply Matrices (12.2)

- Matrix multiplication can only happen if the number of columns of the first matrix is the same as the number of rows on the second matrix.
- You can multiply a 3×5 with a 5×2.
  - 3×**5** · **5**×2 → 3×2 will be the dimensions of the answer
- Because of this **order does matter**!

1-05 Multiply Matrices (12.2)

$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot -2 + 2 \cdot 4 & 1 \cdot 1 + 2 \cdot 3 \\ 0 \cdot -2 + -3 \cdot 4 & 0 \cdot 1 + -3 \cdot 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 7 \\ -12 & -9 \end{bmatrix}$$

1-05 Multiply Matrices (12.2)

$$\begin{bmatrix} 1 & 0 & 4 \\ -2 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot -1 + 0 \cdot 3 + 4 \cdot 5 \\ -2 \cdot -1 + 3 \cdot 3 + 2 \cdot 5 \end{bmatrix}$$

$$\begin{bmatrix} 19 \\ 21 \end{bmatrix}$$

1-05 Multiply Matrices (12.2)

- Use the given matrices to evaluate $2(AC) + B$
- $A = \begin{bmatrix} 5 & -9 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

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$$2\left(\begin{bmatrix} 5 & -9 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \end{bmatrix}\right) + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$
$$2\left(\begin{bmatrix} 5 \cdot 2 + (-9) \cdot (-6) \\ (-1) \cdot 2 + 3 \cdot (-6) \end{bmatrix}\right) + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$
$$2\begin{bmatrix} 64 \\ -20 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} 128 \\ -40 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} 128 \\ -36 \end{bmatrix}$$

1-05 Multiply Matrices (12.2)

- The members of two bowling leagues submit meal choices for an upcoming banquet as shown. Each pizza meal costs $16, each spaghetti meal costs $22, and each Sam's chicken meal costs $18. Use matrix multiplication to find the total cost of the meals for each league.

	Pizza	Spaghetti	Sam's Chicken
League A	18	35	7
League B	6	40	9

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$$\begin{bmatrix} 18 & 35 & 7 \\ 6 & 40 & 9 \end{bmatrix} \begin{bmatrix} 16 \\ 22 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} 18 \cdot 16 + 35 \cdot 22 + 7 \cdot 18 \\ 6 \cdot 16 + 40 \cdot 22 + 9 \cdot 18 \end{bmatrix}$$

$$\begin{bmatrix} 1184 \\ 1138 \end{bmatrix}$$

League A: $1184, League B: $1138

1-06 Evaluate Determinants (12.3)

After this lesson…

- I can find the determinant of a square matrix.
- I can use determinants to find areas of triangles.
- I can use determinants to solve systems of equations.

1-06 Evaluate Determinants (12.3)

- You had to know that all this matrix stuff must have some purpose.

- Uses of matrices (that we will investigate today)
  - Solve systems of equations
  - Find the area of a triangle when we only know the coordinates of its vertices

1-06 Evaluate Determinants (12.3)

- Determinant
  - Number associated with square matrices

  - Symbolized by *det A* or | A |
    - Vertical lines mean determinant
    - I won't answer that question on the test for you!

1-06 Evaluate Determinants (12.3)

- Determinant of 2×2 matrix
  - Multiply along the down diagonal and subtract the product of the up diagonal.

- $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$

$$2(4) - 3(-1) = 8 + 3 = 11$$

1-06 Evaluate Determinants (12.3)

- Determinant of 3×3 Matrix
  - Copy the first 2 columns behind the matrix and then add the products of the down diagonals and subtract the product of the up diagonals.

- $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$

$$\begin{gathered} 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2 \\ = 45 + 84 + 96 - 105 - 48 - 72 \\ = 225 - 225 \\ = 0 \end{gathered}$$

1-06 Evaluate Determinants (12.3)

- Area of a Triangle

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where *x*’s and *y*’s are the coordinates of the vertices

1-06 Evaluate Determinants (12.3)

- Find the area of a triangle with vertices of (2,4), (5,1), and (2,-2)
- $Area = \pm\frac{1}{2}\begin{vmatrix} 2 & 4 & 1 \\ 5 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$

$$\pm\frac{1}{2}\left(2\cdot1\cdot1 + 4\cdot1\cdot2 + 1\cdot5\cdot(-2) - 2\cdot1\cdot1 - (-2)\cdot1\cdot2 - 1\cdot5\cdot4\right)$$

$$\pm\frac{1}{2}(2 + 8 + (-10) - 2 - (-4) - 20)$$

$$\pm\frac{1}{2}(-18)$$

$$9$$

$$Area = 9$$

1-06 Evaluate Determinants (12.3)

- Cramer's Rule
  - Write the equations in standard form
  - Make a matrix out of the coefficients
- 2×2
- $\begin{array}{l} ax + by = e \\ cx + dy = f \end{array}$ gives $x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

Notice that the numerator and denominator are the same except for the columns containing the coefficients of the variable you are solving for is replaced with the numbers from the constants column.

1-06 Evaluate Determinants (12.3)

- $\begin{aligned} 2x + \quad y &= \quad 1 \\ 3x - 2y &= -23 \end{aligned}$

$$x = \frac{\begin{vmatrix} 1 & 1 \\ -23 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}} = \frac{1(-2) - (-23)(1)}{2(-2) - 3(1)} = \frac{21}{-7} = -3$$

$$y = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -23 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}} = \frac{2(-23) - 3(1)}{-7} = \frac{-49}{-7} = 7$$

(−3, 7)

1-06 Evaluate Determinants (12.3)

- Cramer's Rule on a 3×3 System
  - Same as 2×2 system
  - The denominator is the determinant of the coefficient matrix and the numerator is the same only with the column of the variable you are solving for replaced with the = column.

1-06 Evaluate Determinants (12.3)

- $2x - y + 6z = -4$
  $6x + 4y - 5z = -7$
  $-4x - 2y + 5z = 9$

$$x = \frac{\begin{vmatrix} -4 & -1 & 6 \\ -7 & 4 & -5 \\ 9 & -2 & 5 \end{vmatrix} \begin{matrix} -4 & -1 \\ -7 & 4 \\ 9 & -2 \end{matrix}}{\begin{vmatrix} 2 & -1 & 6 \\ 6 & 4 & -5 \\ -4 & -2 & 5 \end{vmatrix} \begin{matrix} 2 & -1 \\ 6 & 4 \\ -4 & -2 \end{matrix}}$$

$$= \frac{(-4)(4)(5) + (-1)(-5)(9) + (6)(-7)(-2) - (9)(4)(6) - (-2)(-5)(-4) - (5)(-7)(-1)}{(2)(4)(5) + (-1)(-5)(-4) + (6)(6)(-2) - (-4)(4)(6) - (-2)(-5)(2) - (5)(6)(-1)}$$

$$= \frac{-162}{54} = -3$$

$$y = \frac{\begin{vmatrix} 2 & -4 & 6 \\ 6 & -7 & -5 \\ -4 & 9 & 5 \end{vmatrix} \begin{matrix} 2 & -4 \\ 6 & -7 \\ -4 & 9 \end{matrix}}{\begin{vmatrix} 2 & -1 & 6 \\ 6 & 4 & -5 \\ -4 & -2 & 5 \end{vmatrix} \begin{matrix} 2 & -1 \\ 6 & 4 \\ -4 & -2 \end{matrix}}$$

$$= \frac{(2)(-7)(5) + (-4)(-5)(-4) + (6)(6)(9) - (-4)(-7)(6) - (9)(-5)(2) - (5)(6)(-4)}{54}$$

$$= \frac{216}{54} = 4$$

$$z = \frac{\begin{vmatrix} 2 & -1 & -4 \\ 6 & 4 & -7 \\ -4 & -2 & 9 \end{vmatrix} \begin{matrix} 2 & -1 \\ 6 & 4 \\ -4 & -2 \end{matrix}}{\begin{vmatrix} 2 & -1 & 6 \\ 6 & 4 & -5 \\ -4 & -2 & 5 \end{vmatrix} \begin{matrix} 2 & -1 \\ 6 & 4 \\ -4 & -2 \end{matrix}}$$

$$= \frac{(2)(4)(9) + (-1)(-7)(-4) + (-4)(6)(-2) - (-4)(4)(-4) - (-2)(-7)(2) - (9)(6)(-1)}{54}$$

$$= \frac{54}{54} = 1$$

(−3, 4, 1)

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

After this lesson…

- I can find the inverse of a matrix.
- I can solve linear systems using inverse matrices.
- I can solve real-life problems using inverse matrices.

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

- You can use matrices to solve linear systems in ways different from Cramer's Rule.

- We will learn how, but it requires that we know how to find inverse matrices.

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

- The Identity Matrix multiplied with any matrix of the same dimension equals the original matrix.
- $A \cdot I = I \cdot A = A$
- This is the matrix equivalent of 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1’s on diagonal, everything else is zero

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

- You cannot divide by a matrix!
- So we multiply by the inverse of a matrix.
- $A \cdot A^{-1} = [1] = I$
    - Just like $x\,(x^{-1}) = x\left(\frac{1}{x}\right) = 1$
- If A, B, and X are matrices, and
    - $A \cdot X = B$
    - $A^{-1} \cdot A \cdot X = A^{-1} \cdot B$
    - $I \cdot X = A^{-1} \cdot B$
    - $X = A^{-1} \cdot B$

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

- The Rule for 2×2 (Memorize)
- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- $$A^{-1} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{1(4) - 3(2)} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$

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$$\begin{bmatrix} -2 & -1 \\ 4 & 0 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} -2 & -1 \\ 4 & 0 \end{vmatrix}} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \frac{1}{-2(0) - 4(-1)} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 0 & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}}$$

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- Solve a matrix equation
  - $AX = B$
  - $\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix} X = \begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$
- Find A^{-1}

$$\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix}^{-1} = \frac{1}{-3(-7) - 5(4)} \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$$

$$= \frac{1}{1} \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}$$

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

- $A^{-1}AX = A^{-1}B$

$$\begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix}X = \begin{bmatrix} -7 & -4 \\ -5 & -3 \end{bmatrix}\begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$$

$$I \cdot X = \begin{bmatrix} -7(3) + -4(2) & -7(8) + -4(-2) \\ -5(3) + -3(2) & -5(8) + -3(-2) \end{bmatrix}$$

$$X = \begin{bmatrix} -29 & -48 \\ -21 & -34 \end{bmatrix}$$

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- Solve a system of linear equations
- $\begin{aligned} 2x + \quad y &= -13 \\ x - 3y &= \quad 11 \end{aligned}$
- Take your equation and write it as matrices
  - $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$
- Find the coefficient matrix inverse

$$\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}^{-1} = \frac{1}{-7} \begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}$$

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

- Multiply the front of both sides by the inverse
  - Left side becomes $\begin{bmatrix} x \\ y \end{bmatrix}$
  - $\begin{bmatrix} x \\ y \end{bmatrix} =$

$$\frac{1}{-7}\begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -13 \\ 11 \end{bmatrix}$$
$$= \frac{1}{-7}\begin{bmatrix} 39 + -11 \\ 13 + 22 \end{bmatrix}$$
$$= \frac{1}{-7}\begin{bmatrix} 28 \\ 35 \end{bmatrix}$$
$$= \begin{bmatrix} -4 \\ -5 \end{bmatrix}$$

(-4, -5)